RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIFTH SEMESTER EXAMINATION, MARCH 2022

THIRD YEAR [BATCH 2019-22]

Date : 26/02/2022 **MATHEMATICS**

Time : 11am-1pm **Paper** : MTMA CC11 Full Marks : 50

Group A

Answer any 4 questions. **[5 x 4 = 20 marks]**

1. Obtain a field with 4 elements by adjoining a root of $x^2 + x - 1$ to the field $\frac{\mathbb{Z}}{2\mathbb{Z}}$. [5]
2. Determine the minimal polynomial over $\mathbb{Q}$ for $1 + i$. [5]
3. Determine the degree of $1 + \sqrt[3]{2} + \sqrt[3]{4}$ over $\mathbb{Q}$. [5]
4. Prove that $\mathbb{Q}(\sqrt{2} + \sqrt{3}) = \mathbb{Q}(\sqrt{2}, \sqrt{3})$. [5]
5. Determine the degree of $\sqrt{5} + \sqrt[3]{7}$ over $\mathbb{Q}$. [5]
6. Determine the minimal polynomial over $\mathbb{Q}$ for $2 + \sqrt{3}$. [5]

Group B

Answer any one from Question Nos.7 to 8 and any two from Question Nos. 9 to 11

7. Use Charnes Big M-method to solve the L.P.P. [6]

$$\begin{aligned} \text{Maximize} \quad & z = 5x_1 - 2x_2 + 3x_3 \\ \text{Subject to} \quad & 2x_1 + 2x_2 - x_3 \geq 2 \\ & 3x_1 - 4x_2 \leq 3 \\ & x_2 + 3x_3 \leq 5 \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$

8. Use the two phase simplex method to solve the L. P. P. [6]

$$\begin{aligned} \text{Minimize} \quad & z = 3x_1 + 2x_2 \\ \text{Subject to} \quad & 2x_1 + x_2 \geq 14 \\ & 2x_1 + 3x_2 \geq 22 \\ & x_1 + x_2 \geq 1 \\ & x_1, x_2 \geq 0. \end{aligned}$$

9. (a) Find out all basic solutions to the following equations identifying in each case the basic vectors , basic and non-basic variables [5]

$$\begin{aligned} 2x_1 - 5x_2 + x_3 + 3x_4 &= 4 \\ 3x_1 - 10x_2 + 2x_3 + 6x_4 &= 12. \end{aligned}$$

(b) Find the optimal solution of the following problem by solving its dual : [7]

$$\begin{aligned} \text{Minimize} \quad & z = x_1 + 2x_2 + 3x_3 \\ \text{Subject to} \quad & x_1 - x_2 + x_3 \geq 4 \\ & x_1 + x_2 + 2x_3 \leq 8 \\ & x_2 - x_3 \geq 2 \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$

10. (a) Prove that $x_1 = 4, x_2 = 1, x_3 = 3$ is a feasible solution to the set of equations

$$\begin{aligned} 2x_1 - 3x_2 + x_3 &= 8, \\ x_1 + 2x_2 + 3x_3 &= 15. \end{aligned}$$

Reduce the feasible solution to basic feasible solutions. [6]

(b) A car hire company has one car in each of the five depots a, b, c, d and e . A customer in each of the five towns A, B, C, D and E requires a car. The distance (in km.) between the depots (origins) and the towns (destinations) where the customers are , given by the following distance matrix :

	a	b	c	d	e
A	20	40	30	50	40
B	70	40	60	80	40
C	20	90	80	100	40
D	80	60	120	70	40
E	20	80	50	80	80

How should the cars be assigned to the customers so as to minimize the distance travelled?

[6]

11. (a) Find an optimal solution and corresponding cost of the following transportation problem

[6]

	D_1	D_2	D_3	D_4	a_i
O_1	6	1	9	3	70
O_2	11	5	2	8	55
O_3	10	12	4	7	90
b_j	85	35	50	45	

(b) Examine analytically whether the following sets X are convex or not. [3 + 3]

i. $X = \{(x_1, x_2) : 2x_1 + x_2 \geq 20, x_1 + 2x_2 \leq 80, x_1 + x_2 \leq 50, x_1, x_2 \geq 0\}$.

ii. $X = \{(x_1, x_2) : 9x_1^2 + 4x_2^2 \leq 36\}$.

– – – – × – – – –